Use of Pressure Management to Reduce the Probability of Pipe Breaks: A Bayesian Approach

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Abstract: As pipe breaks in water distribution networks produce serious consequences, water authorities strive to minimize the frequency of their occurrence. Pressure management is an essential tool to reduce the frequency of breaks and it is closely linked to the proper analysis of a maximum pressure indicator. A methodology that compares the unconditional cumulative distribution function (CDF) and the parametric break-conditioned CDF of the maximum pressure indicator is proposed in this paper. The relationship between the CDFs compared is established by means of the Bayes’ theorem, which allows determining a probability ratio. The objective is to identify the range of operation of maximum pressure that is most likely to reduce pipe breaks. The methodology is applied to four sectors of the water distribution network in Madrid (Spain). In three of those sectors, the maximum pressure indicator is a good predictor of the probability of pipe breaks, confirming that the probability of breaks increases for high maximum pressure ranges. The methodology is validated in one sector, and results provide good agreement between predicted and observed failure rates. DOI: 10.1061/(ASCE)WR.1943-5452.0000519. © 2015 American Society of Civil Engineers.

Author keywords: Pressure management; Pipe breaks; Statistical analysis; Bayes’ theorem.

Introduction

One of the manifestations of the deterioration of water supply systems is frequent pipe breaks. System failures are one of the major problems to be faced by water utilities due to their adverse consequences. Networks may fail to fulfill the required levels of water capacities and pressure heads offered to customers. Additionally, breaks may cause water contamination by the intrusion of polluted water. Other significant social impacts include the reduction in the water-carrying capacity of pipes, service interruptions to end users, flooding of private property, traffic congestion, damage to real estate, loss of amenity due to noise, and disruption of local economic activity.

For all these reasons, water utilities aim at mitigating the problem of break occurrence to as great an extent as possible. Since a large investment is required to renew and replace pipe networks (Gomes et al. 2011), several predictive deterioration models have been developed (Kleiner and Rajani 2001) to assist water utilities in the decision-making process (Wang et al. 2009). Predictive models may be classified into physically based models, statistical models, and data mining methods (Xu et al. 2011). Physical models attempt to elucidate the physical mechanisms behind pipe breaks. Statistical models are based on historical data and intend to reveal significant pipe break correlations. Data mining methods have recently emerged in this context due to the complexity of water network systems and aim at discovering patterns in pipe failures data sets (Berardi et al. 2008). The statistical models allow representing break behavior and patterns in water mains (Wang et al. 2009) based on historical data of deteriorating factors (Kleiner and Rajani 2001). They are considered a cost-effective tool to analyze and assess the probability of pipe failure (Xu et al. 2011). Predicting failures in pipelines is challenging because they depend on many factors that are difficult to characterize quantitatively (Babovic et al. 2002). Pressure management, which is usually related to the reduction of maximum operating pressure (Thornton and Lambert 2007), is considered an effective tool for leakage control (Gomes et al. 2012), and it has also been shown to yield a decrease in the frequency of pipe breaks (Palau et al. 2012; Lambert and Thornton 2011). Pressure management strives to ensure an optimum level of service, while eliminating or reducing pressure transients and large pressure fluctuations. These goals are achieved through fault pressure level controls and pressure reduction protocols (Thornton and Lambert 2006), and is regarded as one of the least costly alternatives to meeting water supply needs (Girard and Stewart 2007). Unfortunately, it is difficult to establish both the causal relationships between pressure and pipe breaks, and the pressure operating ranges that are to be established in order to achieve a reduction in the number of such breaks (Lambert and Fantozzi 2010). In addition, the consequences of the pressure management depend on the conditions of the water distribution systems (Fantozzi and Lambert 2007). Therefore, as water utilities encounter difficulties in making decisions, decision-support tools are required. In addition, this also involves consideration of uncertainties associated with the recorded data, as it provides imprecise results of the methodologies or models used.

To address the above challenges and the imprecision of recorded data, and to guide the decision-making process, some authors have used probability rules (Babovic et al. 2002) and, in particular, the
Bayes’ theorem (Kulkarni et al. 1986; Watson et al. 2004; Watson and Mason 2006; Watson 2005; Economou et al. 2007, 2008; Dridi et al. 2005, 2009; Economou 2010). Babovic et al. (2002) show that the causal relationship between two variables may be established in a probabilistic sense. They also suggest that probability functions may reduce the uncertainty of an underground asset where precise information is difficult to access. Kulkarni et al. (1986) developed a Bayesian diagnostic model to predict the probability of failure in gas pipeline systems. They conditioned the probability of failure to specific system characteristics and proposed that, if it is significantly larger than the system-wide probability of failure, then the set of characteristics may explain a relatively high failure rate (Kleiner and Rajani 2001). Their results indicate that the model adequately represents the observed data but does not consider the time of break occurrence. Watson et al. (2004), Watson and Mason (2006), and Watson (2005) use the Bayes’ theorem to establish a relationship between two variables (number of failures and failure rate), considering observed data, in water distribution systems. They developed a Bayesian-based decision support system to facilitate the identification of efficient asset management policies. The Bayesian approach allows incorporating, expressing, and updating the uncertainty to determine such policies. They argue that it is a suitable tool for use in water distribution networks where the time series of data is limited and when the quality needed for a deterministic analysis is lacking. Economou (2007, 2008), Dridi et al. (2005, 2009), and Economou (2010) follow the Bayesian diagnostic model of Watson et al. (2004), although they propose a different approach to modeling the occurrences of pipe failures in time.

In this paper, a Bayesian diagnostic methodology to link the probability of pipe breaks to operating pressure in water distribution networks is proposed. The research question addressed here is what range of operating pressure should be established in order to reduce the probability of pipe breaks? Operating pressure in a district metered area (DMA) is characterized by means of a pressure indicator at the head of the DMA, which is added to the elevation difference between the entry of the area and the average elevation of the DMA (García et al. 2006). A pressure indicator may be defined as the calculated statistic from the time series of pressure head over a specific time window. If the statistic is the maximum value over the time window, the indicator is referred to as a maximum pressure indicator, which is one chosen in this article. The time window includes an awareness time of pipe breaks, which has been estimated in the literature to be as much as 3 days (Morrison 2004). The proposed methodology is based on a two-step analysis. In the first step, the probability distribution of the hourly time series of the maximum pressure indicator is compared with the probability distribution of indicator values at times coincident with a reported pipe break. If both probability distributions are significantly different, it may be concluded that there is a relationship between the maximum pressure indicator and the occurrence of pipe breaks, and the nature of this relationship is analyzed. In the second step, the ranges of the maximum pressure indicator associated with a higher probability of pipe breaks are identified by means of a Bayesian approach. Finally, the results of the analysis may be used to support pressure-management protocols aimed at efficiently reducing the excess pressures that may cause pipe breaks. The effectiveness of the approach is illustrated by applying the methodology to four sectors of the water distribution network of Madrid (Spain) and by validating it in one sector. The results show that the proposed methodology is a valuable tool to use in managing pressure in order to reduce pipe breaks in water distribution systems.

**Available Data**

**Pressure Head**

Instantaneous pressure values, which are accurate to 1 m, are measured at the head of each DMA every 15 min. The time series of pressure data is periodically transmitted to a control center in order to be processed and stored. The average of four 15-min instantaneous pressure values is calculated to yield the hourly average pressure head. The resulting time series of hourly average pressures is the basis for the present study. It has to be taken into account that the nature of the pressure data made it impossible to determine if the breaks were due to short-term hydraulic transients (Wang et al. 2014) during the 15-min sampling intervals. The pressure head in this study is measured at the head of the DMA. As the head losses are relatively small, the average pressure over the DMA may be estimated by adding to the pressure head measured at the head of the sector the difference between the elevation at the head of the sector and the average elevation of the DMA (García et al. 2006). The average elevation of the different zones and the elevation of each entry point are shown in the “Case Study” section.

**Registry of Pipe Breaks**

In the studied case, the water utility kept a record of pipe breaks. The vast majority of those pipe breaks were identified thanks to user reports. After a pipe break is reported, field work allows determining the exact location where the pipe failed and whether the failure was due to internal or external causes, such as inadequate manipulation. Only unintentional pipe breaks were considered in the present study. Every pipe break is identified by a distinct identification code. The databases record not only the identification code, but also the location of the break and the moment when the incident was first reported. The coordinates of the broken pipe allow locating the break within a specific DMA. The available data for this study are therefore the identification codes of reported failures, the location of the broken pipes, and the time when they were first reported. The water utility did not keep a detailed record of failure modes, and therefore no attempt was made to identify and remove from the analysis those breaks that were not directly related to pressure. General information of the properties of the sectors is also provided and presented below in the section “Case Study.”

**Methodology**

Water utilities do not usually have an entire history of pipe failures, and therefore the available data is often limited to recent periods of time (Watson et al. 2004). A Bayesian model can provide estimates of the probability of pipe breaks based on this type of relatively sparse data (Watson et al. 2006). The present diagnostic Bayesian methodology uses recorded pipe breaks in addition to the time series of pressure at the head of the DMA. The proposed methodology compares estimates of cumulative distribution functions (CDFs) of a pressure indicator through consideration of two situations: under standard operation regimes, and when they are coincident with the occurrence of a pipe break. In the former case, the CDF is named the generic or unconditional CDF of the pressure indicator. In the latter, the CDF is referred to as the break-conditioned CDF of the indicator. Statistically, when the compared samples do not follow the same distribution function, it is concluded that the analyzed indicator is dependent on the probability of pipe breaks. Bayes’ theorem is subsequently invoked in order to determine the pressure indicator ranges that increase the probability of pipe breaks. Although pipes are designed to withstand normal
operating pressures with a safety factor greater than 1, pipes deteriorate with time and become more vulnerable to pressure-related failures. The presented methodology is not an attempt to derive a general relationship between the pressure indicator and the probability of pipe breaks for a distribution system as a whole. In this case, both other causes of deterioration, such as material and age of the pipes, and the mechanism of failures should have been taken into account. Instead, the main objective of the present study is to derive a robust and simple methodology to help water utilities in their difficult task of deciding what pressure limits to impose, in a pressure-managed DMA, in order to meet their dual objective of reducing pipe breaks while providing adequate service to the end users.

The analysis process is summarized in Fig. 1, and comprises the following basic steps: (1) analysis of the pressure indicator, (2) analysis of pressure indicator conditioned to breaks, (3) comparison, and (4) validation. The first and second steps of the methodology entail determining the probability distribution of the pressure indicator unconditioned and conditioned to breaks. In the third step, the unconditional CDF and the break-conditioned CDF are compared to calculate—the through the Bayes’ theorem—the pressure indicator ranges that increase or reduce the probability of pipe breaks. Finally, the validation step provides a quantitative assessment of the predictive power of the proposed methodology.

As shown in Fig. 1, the analysis of the maximum pressure indicator determines the probability distribution of the pressure indicator through its CDF. In the analysis of the indicator conditioned to breaks, the Kolmogorov-Smirnov test (K-S) is applied to compare the empirical break-conditioned CDF and the unconditional CDF. If the test is negative, it is concluded that there is a relationship between the pressure indicator and the probability of breaks \( P(B) \). The K-S test is used to identify the DMAs where the pressure indicator influences the probability of breaks. In addition to the empirical break-conditioned CDF, a parametric break-conditioned CDF is also estimated to avoid problems due to the small number of points of the empirical curve. The Bayesian information criterion (BIC) identifies the most likely parametric CDF that fits the empirical break-conditioned CDF. The Chi-squared test \( (\chi^2)^2 \) test) assesses whether the parametric break-conditioned CDF belongs to the same population as the sample of break-conditioned values of the pressure indicator. The parametric break-conditioned CDF aims at identifying the probability distribution of the pressure indicator conditioned to breaks. In the third step, the probability distribution of the unconditional pressure indicator, and of the pressure indicator conditioned to breaks, are compared through the Bayes’ theorem. The probability ratio (PR) allows the identification of a pressure indicator threshold, so that for larger values of the pressure indicator the probability of a break increases. In the validation period, the predicted and observed failure rates are compared for the entire validation period and when the pressure indicator is above and below the obtained threshold.

**Unconditional CDF of the Pressure Indicator**

The hourly time series of the maximum pressure indicator is derived from the hourly pressure time series by computing, for each time step, the maximum pressure over the specified time window. It is calculated as follows:

\[
I_i = \max_{k=0,1,\ldots,n-1}(p_{i-k})
\]

where \( I_i \) = maximum pressure indicator at time \( i \); \( p_i \) = hourly pressure value at time \( i \); and \( n \) = number of time steps in the window with.

The cumulative distribution function of the pressure indicator and the probability distribution of the indicator are computed from the marginal distribution of the maximum pressure indicator time series.

**Break-Conditioned CDF of the Pressure Indicator**

The break-conditioned time series of the maximum pressure indicator is obtained by selecting, from the hourly time series of the maximum pressure indicator, those values that correspond to an instant when a pipe break was reported within the sector. It

\[
\text{Bayes theorem: probability ratio } PR = \frac{P(I|B)}{P(I)}
\]

\[
\text{Pressure indicator ranges when the probability of breaks increases } P(B/I)
\]

\[
\text{Comparison between predicted and observed failure rates in the validation period}
\]

---

**Fig. 1.** Flowchart of the proposed methodology for reducing pipe breaks in water distribution networks
represents the maximum instantaneous pressure value recorded right before a pipe break was reported, and over a specific window width. The number of pressure indicator values conditioned to breaks is therefore equal to the number of recorded breaks in the sector (as they are calculated before each pipe break). The obtained pressure indicator values are used to estimate the break-conditioned CDF of the pressure indicator.

Fig. 2(a) shows the method to determine a pressure indicator conditioned to breaks. The time series of water pressure and a pipe break for a DMA are shown during the period of September 6–13, 2008. In particular, it shows how the pressure values used in the calculation of the pressure indicator conditioned to breaks are limited. A 24-h window immediately preceding the failure event is selected. The window width limits the pressure values used to calculate a pressure indicator conditioned to breaks. As the awareness time of a larger pipe break could typically be up to 3 days (Morrison 2004), these pressure values are selected within a specific window width.

Fig. 2(b) shows the method to compare both functions: the CDF of the maximum pressure indicator (dots) and the break-conditioned CDF of the same indicator. Pressure indicator values conditioned to breaks allow estimating the empirical break-conditioned CDF (open circles). The parametric break-conditioned CDF (black solid line) represents the best fit to the empirical break-conditioned CDF. It allows calculating the probability distribution of the indicator conditioned to breaks for the same indicator range as the probability distribution of the indicator.

To compare the pair of distribution functions, the unconditional CDF and the empirical break-conditioned CDF, the K-S nonparametric test is applied. In this application, the K-S test assesses whether the two samples come from the same parent population (null hypothesis). Suppose, \( F_1(x) \) and \( F_2(x) \) are two CDFs of two sample data of a variable \( x \). \( F_1(x) \) is the empirical break-conditioned CDF and \( F_2(x) \) is the unconditional CDF. The null hypothesis, \( H_0 \), and the alternative hypothesis, \( H_A \), concerning their CDFs are

\[
H_0: X \rightarrow F_1(x) = F_2(x) \tag{2}
\]

\[
H_A: X \rightarrow F_1(x) \neq F_2(x) \tag{3}
\]

and the test statistic, \( T \), is defined as

\[
T = \max_x |F_1(x) - F_2(x)| \tag{4}
\]

which is the maximum vertical distance between the distributions \( F_1(x) \) and \( F_2(x) \). If the test statistic is greater than some critical value, the null hypothesis is rejected (Khan et al. 2006). If this is case for the two CDFs being compared, a relationship between the pressure indicator and the probability of pipe breaks may be established.

The CDF of the indicator conditioned to breaks could be estimated in a precise manner if the number of years of recorded breaks was long enough. However, the length of the historical time series of breaks is insufficient to guarantee a precise statistical characterization of the extreme indicator values conditioned to breaks from the empirical distribution and this could lead to instabilities in the estimation of the probability of pipe breaks. To overcome this limitation, this study proposes to fit the empirical break-conditioned CDF of the indicator to a parametric CDF. The method used to fit the CDF of the pressure indicator conditioned to breaks to a parametric CDF is based on the BIC. This BIC criterion is defined as follows:

\[
BIC = -2 \log t(L) + t \log(s) \tag{5}
\]

where \( t = \) number of parameters in the model; \( L = \) probability of the fitted model; and \( s = \) sample size (de-Graft Acquah 2010). The most probable model is that which minimizes the BIC criterion.

Subsequently, the Chi-squared test is applied to establish whether the sample of break-conditioned values of the indicator follows the same parametric distribution function used in the fitting. The Chi-squared test contrasts the null hypothesis that considers if a sample comes from a specified distribution function of probability (Chernoff and Lehmann 1954). The null hypothesis, represented by \( H_0 \), and the alternative hypothesis, \( H_A \), may be written as follows:

\[
H_0: X \rightarrow F_1(x) = F_0(x) \tag{6}
\]

\[
H_A: X \rightarrow F_1(x) \neq F_0(x) \tag{7}
\]

where \( X = \) random variable to be analyzed; \( x = \) sample values of the maximum pressure indicator conditioned to breaks; \( F_1(x) = \) empirical break-conditioned CDF and \( F_0(x) = \) the parametric break-conditioned CDF. At any significance greater than the \( p \)-value, the null hypothesis should be rejected.

The best parametric break-conditioned CDF fitted to the empirical CDF is also represented in Fig. 2(b). The parametric CDF

\[
F_{\hat{X}}(x) = F_{\hat{X}}(x) \tag{8}
\]
allows the probability of the maximum pressure indicator conditioned to breaks to be determined.

**Comparison: The Probability Ratio**

Bayes’ theorem is applied in order to establish a relationship between the estimated cumulative probability distribution functions: the distribution function of the pressure indicator and the distribution function of the indicator conditioned to breaks.

Two distinct events are considered for a 1-h time interval $\Delta t$:
- Event $B$: A pipe break occurs in the DMA.
- Event $I$: The pressure indicator takes a value in the interval $[I_\alpha, I_\beta]$.

The probabilities of these two events may be estimated from the available data. The probability of having a pipe break in any time interval of one hour is estimated as the number of breaks registered in the data period, $N_b$, divided by the number of hours in the data period, $N_T$:

$$P(B) = \frac{N_b}{N_T} \quad (8)$$

The probability of the pressure indicator taking a value in the interval $[I_\alpha, I_\beta]$ can be estimated from the CDF of the pressure indicator

$$P(I) = P(I_\alpha \leq I_i < I_\beta) = F_I(I_\beta) - F_I(I_\alpha) \quad (9)$$

where $I_i$ = maximum pressure indicator in time interval $\Delta t$; and $F_I(x)$ = cumulative distribution function of the maximum pressure indicator particularized for pressure value $x$.

The conditional probability of the pressure indicator taking a value in the interval $[I_\alpha, I_\beta]$ if there is a break in the time interval $\Delta t$ can also be estimated

$$P(I|B) = P(I_\alpha \leq I_i < I_\beta|B) = F_{I_b}(I_\beta) - F_{I_b}(I_\alpha) \quad (10)$$

where $F_{I_b}(x)$ is the break-conditioned cumulative distribution function of the maximum pressure indicator particularized for pressure value $x$.

Therefore, Bayes’ theorem may be applied to obtain the probability of having a pipe break in time interval $\Delta t$ when the pressure indicator takes a value in the interval $[I_\alpha, I_\beta]$:

$$P(B|I) = \frac{P(I|B)P(B)}{P(I)} = \frac{F_{I_b}(I_\beta) - F_{I_b}(I_\alpha) N_b}{F_I(I_\beta) - F_I(I_\alpha) N_T} \quad (11)$$

The above equation expresses the probability of having a break in one time interval when the pressure indicator is in the range given by interval $[I_\alpha, I_\beta]$ and can be related to three factors:
- (1) the unconditional probability of the pressure indicator being in the range $[I_\alpha, I_\beta]$ $[P(I)]$ [Fig. 2(b)],
- (2) the probability of the indicator conditioned to breaks being in the range $[I_\alpha, I_\beta]$ $[P(I|B)]$ [Fig. 2(b)], and
- (3) the system-wide average $P(B)$.

If the break-conditioned and unconditional probabilities of the pressure indicator are similar, the pressure indicator has little

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**Fig. 3.** Representation of the four DMAs (sector and pipes)
diagnostic capability for the range \([I_a, I_b]\). In contrast, if the values are different it may be concluded that the probability of breaks is increased or decreased by the pressure regime. Therefore, the probability ratio \(P(I/B)/P(I)\) provides a measure to establish whether the probability of breaks increases for certain ranges of the pressure indicator. This coefficient is henceforth termed the probability ratio.

**Validation**

To validate the model and elucidate its predictive capacity, the proposed methodology is applied to a separate sample of pipe breaks. To this end, the pressure time series is split into calibration and validation periods, and the agreement between the predicted and observed failure rates is evaluated during the validation period.

In the calibration period, the total number of breaks, \(N_b\), and the number of hours in the data period, \(N_T\), are considered, and its ratio represents the hourly system-wide average probability of breaks, \(P(B) = N_b/N_T\).

In the same calibration period, the methodology allows determining pressure indicator ranges where the probability of breaks increases, that is when the probability ratio is greater than 1. These ranges yield a pressure indicator threshold, which corresponds to the first range that crosses the ratio \(PR = 1\). The first step of the validation process consists in comparing the probability of breaks when the pressure indicators are higher or smaller than this threshold. The probabilities of breaks for different pressure indicator ranges are

\[
[P(B)]_{\text{above}} = \frac{N_b}{N_T}
\]

\[
[P(B)]_{\text{below}} = \frac{N_b}{N_T}
\]

where \(N_b\) and \(N_T\) are the number of breaks where the pressure indicator is above and below the threshold in the validation period, respectively, and \(N_T\) and \(N_T\) are the number of hours when the pressure indicator measurements are above and below the same threshold, respectively.

If the probability of breaks indeed increased when the pressure indicator is above the threshold, then its probability, \([P(B)]_{\text{above}}\), would be greater than the probability of breaks when the pressure indicator is below this threshold, \([P(B)]_{\text{below}}\).

The obtained probabilities in the calibration period are used to calculate the expected number of pipe breaks in the validation period. The expected number of failures may be determined as follows:

\[
E(N_b) = P(B) \cdot N_T
\]

The expected number of pipe breaks in the validation period is represented by \(E(N_b)\), and \(E(N_T)\) is the total number of hours in the validation period. \(E(N_b)\) and \(E(N_T)\) are the expected number of pipe breaks in the validation period, considering the number of hours where the pressure indicator is above, \(N_T\), and below, \(N_T\), respectively. The predicted and observed failure rates are compared to validate the presented methodology.

In the validation period, the performance of the proposed methodology may be assessed by comparing the probability of breaks when the indicator is above the threshold, \([P(B)]_{\text{above, validation}}\), with the average probability of breaks over the whole validation period, \([P(B)]_{\text{validation}}\). The ratio between these two probabilities is named information ratio \(IR\) and has the following expression:

\[
IR = \frac{[P(B)]_{\text{above, validation}}}{[P(B)]_{\text{validation}}}
\]

Information ratios above 1 indicate that pressure management strategies based on the proposed methodology may be successful in reducing the number of pipe breaks.

**Case Study**

The methodology is applied to four sectors or DMAs of a water distribution network located in Madrid (Spain). Fig. 3 shows the representation of the four DMAs. The validation processes is carried out in the first DMA. Table 1 presents the general characteristics of the studied sectors.

Data pertinent to water pressure and pipe breaks were collected from the four DMAs. In addition, pressure data were grouped by periods of time with steady supply conditions and break records were only attached to pipes in the DMA. Fig. 4 shows the number of pipe breaks by DMA and when they were recorded, and also the period of steady pressure heads. The period of time of the third and fourth DMAs with only 17 and 16 breaks are not considered, as it may not represent a reliable way the break-conditioned CDF of the pressure indicator. Pressures are measured at the entry point of the DMA. The average pressure over the DMA is obtained by adding the difference between the elevation of the entry point to the DMA and the average elevation of the section to the pressure at the head of the DMA (Garcia et al. 2006). Table 2 shows the elevation of the entry points to the DMAs and their average elevation, as well as the time period of date records when the number of breaks is greater than 30. The average pressure at the head of the DMA is calculated as the average of the instantaneous pressure values recorded at the head of the DMA and over the period of time shown in Table 2. The average pressure in the DMA is also determined by means of the elevation difference. Table 3 shows the number of pipe breaks, the break rate data, and the properties of the breaks according to pipe material and diameter, for the four DMAs. Unfortunately, records of age and mechanism of failure are not available. Break rates are defined as the number of pipe breaks per year and kilometer of the network in the DMA (number of days year\(^{-1}\) km\(^{-1}\)). As can be seen in Table 3, a high percentage of breaks correspond to fiber-cement pipes although they do not represent the predominant material in the area (Table 1). As for the diameter of the pipes breaks, the percentage of breaks with diameter of less than 150 mm is higher than the percentage of pipe length for this cohort in all cases. The data suggest a relationship between pipe breaks and pipe material and diameter. It may be useful to segregate the break data by homogenous groups of materials, diameter, ages, and mechanism of failure. However, the number of available pipe breaks is insufficient to obtain a robust statistical analysis and a proper estimate of the CDF conditioned to breaks in each cohort. Accordingly, all pipes in every DMA were grouped together in the present analysis. The maximum pressure indicator is chosen in this study, as it is well established that maximum pressure is a key control parameter to understand and reduce the number of pipe breaks in pressure management.
Results and Discussion

**Probability Ratios**

The maximum window width used to determine the maximum pressure indicators is 5 days and is chosen to yield the largest difference between the unconditional CDF and the empirical break-conditioned CDF (Martínez-Codina et al. 2013). The window width of 5 days covers the awareness time for larger pipe breaks that may be up to 3 days (Morrison 2004), allowing for a maximum time error of 2 days.

![Fig. 4. Periods of time with steady conditions of water supply and number of breaks for the four DMAs](image)

The time series of the maximum pressure indicator was used to derive the empirical CDF for the unconditional and break-conditioned cases. The K-S nonparametric goodness of fit test was used to compare both distributions. Table 4 shows the results of the K-S nonparametric test at 95% significance level for the four DMAs. The K-S test demonstrates that the maximum pressure indicator may not influence the probability of breaks for the third DMA because the p-value is above 0.05 (95% confidence level). For the other DMAs, the null hypothesis of the K-S test is rejected. Therefore, the maximum pressure indicator may be studied in more detail in order to determine indicator ranges associated with an increase in the break probability, as this indicator exhibits a distinct statistical behavior when conditioned to breaks.

The break-conditioned CDF was then fitted to a parametric distribution, and the BIC was used to select the model that best fits the break-conditioned CDF of the maximum pressure indicator. The parametric distribution functions tested in the fitting are beta, exponential, extreme value, gamma, generalized extreme value, generalized Pareto, normal, lognormal, Rayleigh, and Weibull. Table 5 indicates that p-values are found above 0.05 for all DMAs. This analysis indicates that all the three parametric distribution functions reproduce quite well the distribution of the maximum pressure indicator conditioned to breaks, at a confidence level of 95%.

![Fig. 5](image)

Fig. 5 shows comparative graphical plots of (1) the unconditional CDF, (2) the empirical break-conditioned CDF, and (3) the parametric break-conditioned CDF, for the maximum pressure indicator and the three selected DMAs. For a specific pressure
interval \([I_a, I_b]\) the CDFs yield the probability of the indicator belonging to the interval. CDF (1) is used for unconditional distribution and CDF (3) for break-conditioned distribution. The relationship between them is referred to as the probability ratio.

In each DMA, the total range of the pressure indicator was divided into six intervals of equal length and the unconditional and break-conditioned probabilities were computed for each interval. The probability ratios \((PRs)\) are shown in Fig. 6, for all three DMAs. The \(PRs\) calculated with the empirical break-conditioned CDF (dashed lines) are distinguished from those determined with the parametric break-conditioned CDF (solid lines). A horizontal line identifies the unit \(PR\). The probability ratio is larger than 1 for certain maximum pressure ranges and, therefore, the probability of failure increases for these indicator ranges. The implication from a management perspective is clear: these latter cases should be avoided in order to minimize the number of breaks in the studied DMAs.

Qualitatively, all \(PRs\) follow a common trend (Fig. 6): the \(PRs\) are lower for small values of the maximum pressure indicator and increase for high values. This means that the probability of breaks increases for high ranges of the maximum pressure indicator. These results suggest that the reduction of the high maximum pressure range is advised.

### Table 2. Date Records, Elevation at the Head of Each DMA, Average Elevation of the DMA, Elevation Difference between the Elevation Head and the Average Elevation, Average Pressure at the Head of the DMA, and the Average Pressure of the DMA Determined by Means of the Elevation Difference for the Four DMAs

<table>
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<th>Date records</th>
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<th>DMA average elevation (m)</th>
<th>Elevation difference (m)</th>
<th>Average pressure at head (m)</th>
<th>DMA average pressure (m)</th>
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### Table 3. Number of Pipe Breaks, Failure Rate, and Number of Breaks by Material and Diameter for the Four DMAs

<table>
<thead>
<tr>
<th>DMA</th>
<th>Number of breaks</th>
<th>Failure rate (breaks/year/km)</th>
<th>Breaks by material</th>
<th>Diameter (mm)</th>
<th>Number</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Fiber cement</td>
<td>150–200</td>
<td>20</td>
<td>43</td>
</tr>
<tr>
<td>1</td>
<td>46</td>
<td>0.57</td>
<td>Ductile iron</td>
<td>ND</td>
<td>7</td>
<td>15</td>
</tr>
<tr>
<td>2</td>
<td>43</td>
<td>0.23</td>
<td>Grey iron</td>
<td>ND</td>
<td>7</td>
<td>15</td>
</tr>
<tr>
<td>3</td>
<td>48</td>
<td>0.53</td>
<td>Polyethylene</td>
<td>ND</td>
<td>7</td>
<td>15</td>
</tr>
<tr>
<td>4</td>
<td>32</td>
<td>0.30</td>
<td>Polyethylene high density</td>
<td>ND</td>
<td>7</td>
<td>15</td>
</tr>
</tbody>
</table>

Note: ND = not defined; PVC = polyvinyl chloride.

### Table 4. Test Results \((p\)-Values\) of the K-S Test for the Difference between Unconditional CDF of the Maximum Pressure Indicator and Empirical Break-Conditioned CDF of the Same Indicator at the 95% Confidence Level for the Four DMAs

<table>
<thead>
<tr>
<th>DMA</th>
<th>(p)-value</th>
<th>K-S test (accepted)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0114</td>
<td>No</td>
</tr>
<tr>
<td>2</td>
<td>0.0126</td>
<td>No</td>
</tr>
<tr>
<td>3</td>
<td>0.0962</td>
<td>Yes</td>
</tr>
<tr>
<td>4</td>
<td>(1.77 \times 10^{-4})</td>
<td>No</td>
</tr>
</tbody>
</table>

### Table 5. For the Selected DMAs, Models Fitted to the CDFs of the Maximum Pressure Indicator Conditioned to Breaks by the BIC Criterion, Their CDF Parameters, and the Results of the BIC Criterion

<table>
<thead>
<tr>
<th>DMA</th>
<th>Parametric CDF</th>
<th>CDF Parameters</th>
<th>BIC (p)-value</th>
<th>Chi-squared test (accepted)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Extreme value</td>
<td>Location Scale</td>
<td>81.0973</td>
<td>126.0296</td>
</tr>
<tr>
<td>2</td>
<td>Gamma</td>
<td>Shape Scale</td>
<td>3.707 \cdot 10^3</td>
<td>138.7333</td>
</tr>
<tr>
<td>3</td>
<td>Extreme value</td>
<td>Location Scale</td>
<td>71.5008</td>
<td>98.6187</td>
</tr>
</tbody>
</table>

Note: The test results \((p\)-values\) of the Chi-squared test for the difference between empirical and parametric break-conditioned CDF of the maximum pressure indicator at the 95% confidence level are also presented.
values may reduce the frequency of breaks in the three selected DMAs.

The crossover thresholds of the maximum pressure indicator, that is when the PR is equal to 1, may be determined from Fig. 6. The parametric distribution function allows smooth intermediate probabilities at the extreme of the indicator conditioned to breaks. For this reason, the thresholds, from which the probability of breaks increases, are calculated from the parametric break-conditioned CDF. These thresholds are 79, 96, and 70 m pressure head for the first, second, and fourth DMA, respectively. It is interesting to note that the proposed strategy imposes these limits on the maximum pressure in order to reduce the number of pipe breaks. As pipes age and deteriorate, their conditions may change, so the estimations of the CDFs need to be periodically repeated to incorporate the changing conditions of the network. Consequently, the obtained thresholds should be updated and it is recommended that they should be used only for short-term planning. Moreover, the difference between thresholds should be noticed. The pressure threshold of the first DMA is 79 and its failure rate is 0.57, whereas the recommended threshold for the second DMA is 96 and its failure rate is 0.23. For DMA 4 the threshold is 70 m and the break rate is 0.30. Differences in behavior can be explained in terms of the different composition of pipe material and diameter and different pressure ratings. Every DMA has specific conditions, like its operating pressure, and pipes are designed based on these conditions, choosing different materials. Therefore, the thresholds obtained are local and valid only for the DMA where the inference was made.

As Fig. 6 shows, the system-wide average probability of breaks, \( P(B) \), could be multiplied by high values for certain ranges of the maximum pressure indicator. The PRs reach values of 2.29, 6.02, and 2.27 if the empirical break-conditioned CDF is considered and 1.78, 4.43, and 2.14 if the parametric break-conditioned CDFs are used, for the three selected DMAs, respectively. The maximum PR is higher when the empirical break-conditioned CDF is considered. As explained previously, when the empirical break-conditioned CDF is fitted to a parametric CDF the intermediate values are smoothed. The obtained PRs for certain indicator ranges confirm that high maximum pressure ranges may allow determining the probability of pipe breaks in this study. As these PRs acquire considerably high values for certain maximum pressure ranges, such ranges should be avoided as far as possible to reduce the probability of pipe breaks.

Validation and Discussion

The proposed methodology is validated on one set of data of the first DMA. The calibration period starts on January 17, 2011 and finishes on January 13, 2012 and 34 breaks occurred in this period. The validation period is from January 13, 2012 to 3 June 3, 2012 and the number of observed breaks is 12. The methodology is applied to the calibration period and the results show that, indeed, the probability of breaks increases when the maximum pressure head indicator threshold is greater than
Table 6. Results of the Validation Processes

<table>
<thead>
<tr>
<th>Period</th>
<th>Formulation</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calibration</td>
<td>Threshold</td>
<td>80.66</td>
<td>m</td>
</tr>
<tr>
<td></td>
<td>( N_b )</td>
<td>30</td>
<td>Number</td>
</tr>
<tr>
<td></td>
<td>( N_T )</td>
<td>8,618</td>
<td>Number</td>
</tr>
<tr>
<td></td>
<td>( P(B) = \frac{N_b}{N_T} )</td>
<td>0.0035</td>
<td>Probability</td>
</tr>
<tr>
<td></td>
<td>( \langle N_b \rangle_{above} )</td>
<td>14</td>
<td>Number</td>
</tr>
<tr>
<td></td>
<td>( \langle N_T \rangle_{above} )</td>
<td>2,189</td>
<td>Number</td>
</tr>
<tr>
<td></td>
<td>( \langle P(B) \rangle_{above} = \frac{\langle N_b \rangle_{above}}{\langle N_T \rangle_{above}} )</td>
<td>0.0064</td>
<td>Probability</td>
</tr>
<tr>
<td></td>
<td>( \langle N_b \rangle_{below} )</td>
<td>16</td>
<td>Number</td>
</tr>
<tr>
<td></td>
<td>( \langle N_T \rangle_{below} )</td>
<td>6,429</td>
<td>Number</td>
</tr>
<tr>
<td></td>
<td>( \langle P(B) \rangle_{below} = \frac{\langle N_b \rangle_{below}}{\langle N_T \rangle_{below}} )</td>
<td>0.0025</td>
<td>Probability</td>
</tr>
<tr>
<td>Validation</td>
<td>( \langle N_b \rangle_{validation} )</td>
<td>12</td>
<td>Number</td>
</tr>
<tr>
<td></td>
<td>( \langle N_T \rangle_{validation} )</td>
<td>3,221</td>
<td>Number</td>
</tr>
<tr>
<td></td>
<td>( \langle N_b \rangle_{above, validation} )</td>
<td>11</td>
<td>Number</td>
</tr>
<tr>
<td></td>
<td>( \langle N_T \rangle_{above, validation} )</td>
<td>2,153</td>
<td>Number</td>
</tr>
<tr>
<td></td>
<td>( \langle N_b \rangle_{below, validation} )</td>
<td>1</td>
<td>Number</td>
</tr>
<tr>
<td></td>
<td>( \langle N_T \rangle_{below, validation} )</td>
<td>1,068</td>
<td>Number</td>
</tr>
<tr>
<td></td>
<td>( \langle E[N_b] \rangle_{validation} = P(B) \cdot \langle N_T \rangle_{validation} )</td>
<td>11.27</td>
<td>Number</td>
</tr>
<tr>
<td></td>
<td>( \langle E[N_b] \rangle_{above, validation} = \langle P(B) \rangle_{above} \cdot \langle N_T \rangle_{above, validation} )</td>
<td>13.78</td>
<td>Number</td>
</tr>
<tr>
<td></td>
<td>( \langle E[N_b] \rangle_{below, validation} = \langle P(B) \rangle_{below} \cdot \langle N_T \rangle_{below, validation} )</td>
<td>2.67</td>
<td>Number</td>
</tr>
</tbody>
</table>

Table 6 summarizes the results of the validation processes. As can be expected, it is relevant to note that the probability of breaks is higher when the pressure indicator is above the threshold than when it is below. It should be mentioned that the expected number of pipe breaks are very close in value to the observed failure rates. The number of pipe breaks in the validation processes is 12 and the expected number is 11.27. If only indicators conditioned to breaks higher or lower than the threshold are considered, the predicted number of breaks, 13.78 and 2.67, is slightly higher than the number of observed failures, 11 and 1. Therefore, these results show good agreement between the predicted and observed pipe breaks.

The IR in the validation period, referred to number of values when the pressure indicator is greater than the threshold, may be calculated as

\[
IR = \frac{\langle P(B) \rangle_{above, validation}}{\langle P(B) \rangle_{validation}} = \frac{\langle E[N_b] \rangle_{above, validation}}{\langle E[N_b] \rangle_{validation}} = 1.37
\]

Consequently, the probability of breaks when the pressure is above the threshold is larger than the probability of breaks in the validation period. In addition, an information ratio above one suggests that the proposed methodology is able to identify pressure ranges linked to higher probability of failure.

However, there may be an optimum threshold that establishes a larger difference between these two probabilities. Fig. 7 shows the IR for different maximum pressure thresholds. As can be seen, the predicted IR obtained with the methodology, 1.37, is close in value to the empirical optimum IR, 1.59. The discretization of the maximum pressure range may affect the results as the number of divisions considered to calculate the predicted IR may vary. In the presented case, the number of divisions is six, and this could explain the slight difference between the predicted IR and the empirical optimum IR.

Although the results show that the proposed analysis can be used to incorporate the goal of reducing pipe breaks while deciding maximum pressure target in pressure management for water distribution networks, there are some limitations. First, the pressure data are limited. Pressure measurements were only collected at 15-min intervals, and therefore the nature of the pressure data made it impossible to determine whether the breaks were due to short term hydraulic transients during the 15-min sampling intervals. The occurrence of pressure transients, which are the cause of a significant fraction of pressure breaks, could not be explicitly included in the pressure indicator used in the analysis. Second, the data on pipe breaks are also limited, because they did not identify failure mode. No attempt was made to remove breaks from the history where the breaks were most likely not due to pressure, as such data were not available. Breaks that were not caused by pressure conditions introduce noise in the data that prevents from obtaining a clear and coherent signal in the analysis. Third, the analysis is based on pressure as the only explanatory variable for pipe breaks. Break probability is a complex function of many other variables, like age, material, or diameter that could have been included in the analysis. The reduced number of pipe breaks in the DMAs under study prevented a more detailed analysis segregating the data by age, material or diameter, as the resulting number of breaks in each cohort would be too small for statistical inference. Fourth, the analysis of pressure is highly simplified. The somewhat arbitrary nature of the adopted pressure indicator is acknowledged. The authors have
worked under the assumption that the average pressure over the entire DMA is representative of pressure supported by every pipe in the area. Head losses and differences in elevation introduce uncertainties that may weaken the correlation between the pressure indicator and pipe working conditions. Fifth, the assessment does not consider dynamic factors. The assumption that the conclusions drawn from breaks registered in a given time period will be valid in the future is clearly flawed because many dynamic factors, like pipe aging, partial renovations, or changes in management, are in place. The analysis would have to be periodically updated to account for the changing conditions in the DMA. Despite these limitations, the analysis advances the knowledge of pipe break behavior in the DMA and provides a methodology for adopting pressure management decisions that may reduce break rate.

Conclusions

This paper presented a new methodology that aims at determining pressure indicator ranges that quantify the probability of pipe breaks in water distribution networks. The proposed methodology identifies a PR that compares the probability distribution of the pressure indicator conditioned to breaks with the unconditional probability distribution of the same indicator. The first one represents the CDF of the pressure indicator conditioned to pipe breaks. The second probability distribution is based on the CDF of the overall pressure indicator values. The PR shows that, when it is greater than 1, the probability of breaks increases for certain pressure ranges. In the case of the PR not exceeding the value of 1, the probability of breaks would not necessarily increase for any values of the pressure indicator.

The methodology, based on the maximum-pressure indicator, was applied to four DMAs in Madrid (Spain), considering periods of time with steady supply conditions and a considerable number of reported breaks. The probability of pipe breaks is calculated through the analysis of the distribution of past failure records. The Kolmogorov-Smirnov nonparametric goodness-of-fit test indicates that the maximum pressure indicator influences the probability of pipe breaks in the first, second, and fourth DMAs, which are selected to apply the methodology.

As the number of points of the empirical break-conditioned is not high, the empirical break-conditioned CDF is fitted to the best parametric distribution function by means of BIC. Test results of the Chi-squared test shows that the parametric CDF can reproduce the distribution of the indicator conditioned to break at the 95% confidence level, for all three selected DMAs.

The results of this study show that the maximum pressure indicator should have an upper limit to reduce the probability of pipe breaks. The thresholds of the maximum pressure indicator in the analyzed sectors are 79, 96, and 70 m for the three sectors examined. These thresholds are obtained when the PR is greater than 1. As pipes age and deteriorate, these thresholds need to be updated with the model and the obtained results may change in time. In addition, the needs of the users of the water distribution systems should be satisfied, such that sufficient water pressure must be provided to the built environment. The maximum water pressure should, at least, be limited to the thresholds that correspond to the highest PR values, which are 1.78, 4.43, and 2.14. Another reason to limit the thresholds to the highest PR values is that pipes are designed to withstand normal operating pressures with a factor of safety greater than 1. The high values of the PR indicate that the maximum pressure range may have diagnostic capability to predict the probability of breaks in this study.

The methodology is validated in the first DMA. In the calibration period, 34 breaks were registered and in the validation period, 12. The calibration period establishes a threshold of the maximum pressure indicator of 80 m, from which the probability of breaks increases. The expected number of breaks calculated with the methodology is 11.27 for all validation period, 13.78 if the pressure indicator is above the calculated threshold and 2.67 if the indicator is below it. The observed number of breaks is very close to the expected number: 12 pipe breaks have been recorded in all validation period, 11 when the indicator is above the threshold and only one when it is below the threshold. The quality of the predictions obtained from the analysis is satisfactory because the results report good agreement between predicted and observed number of failures.

The IR, expressed as the relationship between the probability of breaks when the indicator is above the obtained threshold and the probability of breaks over the entire validation period, is 1.37, close in value to the empirical optimum IR, which is 1.59. Therefore, the methodology represents adequately the difference between the probability of breaks and the probability of breaks conditioned to the maximum pressure indicator.

The proposed methodology could help water utilities reduce the number of system failures through pressure management. The Bayesian approach presented here may be applied to other water distribution networks, and suitable water pressure indicators related to pipe breaks may be explored and tested.

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